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Rapidity-Dependent Low-p_t Enhancement

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Abstract

The rapidity dependence of the low-p_t enhancement is shown to be a sensitive measure of the longitudinal source size for longitudinally expanding finite systems.

Low-p_t enhancement (LPTE) is conventionally understood as a rise in the ratio of the transverse momentum distribution of the particles produced in proton-nucleus $(p + A)$ as well as heavy ion $(A + B)$ reactions as compared to the transverse momentum distribution of the same particles in hadron-hadron reactions at the same center of mass energy per bombarding particle. An up-to-date summary of the status of the experimental data and theoretical understanding of LPTE can be found in ref. [1] which concluded: " In order to understand the results, theoretical models should ... explain the rapidity dependence observed in the data."

In a recent paper the invariant momentum distribution (IMD) and the Bose-Einstein correlation function (BECF) has been calculated for longitudinally expanding finite systems [2].

As an application of those results it is shown here that the longitudinal expansion together with the finite longitudinal size of the expanding tube naturally leads to a *rapidity-dependent* LPTE.

Resonance decay effects [4,5] and other, more exotic sources of LPTE, like the decay of droplets of Quark-Gluon Plasma [6], or the effects of attractive potentials [7] leading to a change in the dispersion relation shall modify primarily the transverse momentum distribution. However, these modifications are due to a mechanism which is independent (modulo isospin) of the longitudinal expansion thus the rapidity dependence of the LPTE does not naturally follows from these models. Partial thermal equilibrium due to a longitudinal expansion was shown to lead to additional soft pions in ref. [8]. These models explain neither the point why the enhancement is similar in $p + A$ and $A + A$ collisions nor the observed rapidity dependence of the LPTE data.

Instead of studying the p_t spectrum directly, let us concentrate on the coupling between the transverse momentum spectra and the *longitudinal* expansion.

Longitudinally expanding finite systems. High energy heavy ion collisions may create one-dimensionally expanding systems, especially in the case of light projectiles. Heavier projectiles may create three-dimensionally expanding systems. However, the three-dimensional expansion does not alter the coupling between transverse momentum spectra and rapidity distribution in the limit $m_t \to m$ as long as the expansion remains in the same limit longitudinally boost-invariant.

The four-momentum is given by $p = (E, \vec{p}) = (E, p_x, p_y, p_z)$. The particle is on the mass shell $m^2 = E^2 - \vec{p}^{\,2}$, the transverse mass is denoted by $m_t = \sqrt{E^2 - p_z^2}$, the rapidity by $y = 0.5 \ln \left(\dfrac{E + p_z}{E - p_z} \right)$, the space-time rapidity by $\eta = 0.5 \ln \left(\dfrac{t + r_z}{t - r_z} \right)$ and the transverse momentum is indicated by $p_t = \sqrt{p_x^2 + p_y^2}$.

If a locally thermalized relativistic momentum distribution creates the correlations between rapidity and space-time rapidity, the one-particle IMD can be calculated for arbitrary

space-time distribution functions as

$$\frac{d^2n}{dydm_t^2} = g(m_t) \int_{-\infty}^{\infty} d\eta \; G(\eta) \, J_{m_t}(\eta, y). \tag{1}$$

In this equation the transverse and temporal components of the space-time distribution have already been integrated over [2], and a finite size in the space-time rapidity is introduced by the distribution function $G(\eta)$. The various rapidity-independent modifications of the low-p_t spectrum are taken into account through the factor $g(m_t)$ which shall be discussed later. Moreover, the local relativistic Bose-Einstein or Fermi-Dirac distributions for longitudinally expanding systems are given by

$$J_{m_t}(\eta, y) \, d\eta = \frac{f}{(2\pi)^3} \frac{d\sigma(\eta) \cdot p}{\exp\left(p \cdot u(\eta)/T\right) \pm 1}. \tag{2}$$

Here f is the degeneracy factor, $d\sigma(\eta) \cdot p$ is the inner-product of the volume-element of the freeze-out hypersurface and the four-momentum $p = (\, m_t \cosh(y),\, p_x,\, p_y,\, m_t \sinh(y) \,)$, while $u(\eta)$ stands for the four-velocity of the expanding matter on the freeze-out hypersurface and T is the freeze-out temperature. The $+$ sign stands for fermions, the $-$ sign for bosons.

For the four-velocity of the 1D expanding matter we assume the scaling Bjorken profile [3] as

$$u(\eta) = (\cosh(\eta),\, 0,\, 0,\, \sinh(\eta)). \tag{3}$$

Accordingly, $J_{m_t}(\eta, y)$ becomes a function of $\eta - y$ only.

All the conventional and non-conventional modifications of the m_t distribution which are independent of the rapidity and aim at describing the low-p_t behaviour at mid-rapidity, enter into the function $g(m_t)$. This function has influence on the rapidity dependence of the effective temperature. We assume that

$$g(m_t) = C \exp(-m_t/T_*) \tag{4}$$

where T_* stands for a *rapidity-independent* effective temperature in the low-p_t limit. Such a structure is the simplest limiting distribution of many complex calculations attempting to

describe the LPTE. Note that here we are interested only in the low-p_t behaviour and thus a deviation of the $g(m_t)$ distribution from the above simple exponential for medium or high values of m_t is not relevant for our considerations. Thus, the phenomenological parameter T_* stands for all the possible rapidity independent modifications of the m_t distribution at low-p_t, including decays of resonances, possible modifications of the dispersion relations and possible more exotic effects.

In this model, the source at a given space-time – rapidity η is emitting bosons with the mean rapidity $\langle y \rangle = \eta$. The width of the emission can be calculated by approximating the Bose-Einstein and the Fermi-Dirac distributions with the relativistic Boltzmann-distribution,

$$J_{m_t}(\eta, y) \approx C'(m_t) \, \exp\left(-m_t (y-\eta)^2/T\right) \tag{5}$$

where $C'(m_t) \equiv \; C \, m_t \, \exp(-m_t/T)$ is independent of rapidity and can thus be absorbed into the $g(m_t)$ factor. Only the leading order $m_t(\eta-y)^2/T$ term has been kept in the Taylor expansion of the logarithm of J_{m_t}. Terms of $\mathcal{O}\left((\eta-y)^2\right)$ are next to leading order since the Boltzmann approximation is valid for $m_t/T >> 1$ only. A numerical comparison of the approximated to its approximation indicates that the approximation is excellent for $m_t > 3T$ and reasonable down to $m_t > 1.2T$. For certain classes of the models [9,10] the freeze-out temperature T is smaller than the effective temperature at mid-rapidity, $T_* = T + T_G$. Here the geometrical contribution to the effective temperature is given in the low-p_t limit by $T_G = m R_G^2 a^2/\tau_0^2$ where the finite transverse geometrical size is R_G, the mean freeze-out time is denoted by τ_0 and a stands for the flow gradinent in units of the mean freeze-out time. A preliminary analysis of the pion IMD [11] indicates that $m_\pi/T \approx 1.5$, for kaons $m_K/T \approx 6$, thus the approximation (5) is warranted.

A Gaussian approximation is also applied for the distribution function of η

$$G(\eta) = \frac{1}{(2\pi\Delta\eta^2)^{1/2}} \, \exp\left(-\frac{(\eta - y_0)^2}{2\Delta\eta^2} \right), \tag{6}$$

$$J_{m_t}(\eta, y) = \frac{1}{(2\pi\Delta\eta_T^2(m_t))^{1/2}} \exp\left(- \frac{(\eta - y)^2}{2\Delta\eta_T^2(m_t)} \right), \tag{7}$$

where

$$\Delta\eta_T\,(m_t) = \sqrt{T/m_t}, \tag{8}$$

the mid-rapidity is denoted by y_0 and the width of the space-time rapidity distribution is given by $\Delta\eta$, which is a dimensionless measure of the longitudinal extension of the expanding system at freeze-out. By introducing this finite width $\Delta\eta$, we break the boost-invariance of our source in the longitudinal direction too. Thus we expect a non-stationary rapidity distribution. The flat rapidity distribution corresponds to the $\Delta\eta \to \infty$ limit.

Gaussian results. The IMD can be calculated [2] as

$$\frac{d^2n}{dydm_t^2} = \frac{1}{(2\pi\Delta y^2(m_t)\,)^{1/2}}\; g(m_t)\;\; \exp(-\frac{(y-y_0)^2}{2\Delta y^2(m_t)}), \tag{9}$$

where

$$\Delta y^2(m_t) = \Delta\eta^2 + \Delta\eta_T^2(m_t). \tag{10}$$

This relation introduces a coupling between the variables y and m_t, i.e. their distributions do not factorize any more. At a fixed value of the rapidity, let us expand eq. (9) around $m_t = m$ as

$$\frac{d^2n}{dy\,dm_t^2} \propto \exp\left(-\frac{m_t}{T_e(y)} + \mathcal{O}\left((m_t - m)^2/m^2\right)\right) \tag{11}$$

which yields a Lorenzian effective temperature distribution in the low-p_t region:

$$T_e(y) = \frac{T_*}{1 + a(y-y_0)^2}, \tag{12}$$

$$a = \frac{T_* T}{2m^2}\left(\Delta\eta^2 + \frac{T}{m}\right)^{-2}, \tag{13}$$

a dependence which is similar to the recent finding of the NA35 collaboration for the charged hadrons in $S + Pb$ collisions at CERN SPS [12]. Such a change in the effective temperature with rapidity is a general property of heavy ion reactions performed both at CERN SPS and at the AGS, as measured by the NA35, E802/E859, E810, E814 collaborations valid pions, kaons protons and lambdas, see Fig. 3. in ref. [13]. Thus the decrease of $T_e(y)$ in the target and projectile fragmentation region can be considered as a simple consequence of both the longitudinal expansion in the final stage and the m_t dependent width of the local thermal rapidity distribution. Simple kinematics results in a decrease of the average transverse momentum in the target and projectile fragmentation regions for $p+p$ reactions too, see Fig. 9 of ref. [14].

With $T_e(y)$ given above the IMD can be integrated over m_t at a fixed y, resulting in

$$\frac{dn}{dy} = \mathcal{N}\, \frac{T_e(y)}{T_*}\, \frac{m + T_e(y)}{m + T_*}\, \exp\left(- \frac{(y - y_0)^2}{2\left(\Delta\eta^2 + T/m\right)} \right), \tag{14}$$

where $\mathcal{N}$ is a normalization constant. This distribution describes for small values of a an approximately Gaussian rapidity distribution, with corrections which reduce the effective width of the Gaussian to $\Delta y_{eff}^2 < \Delta\eta^2 + \frac{T}{m}$.

Eqs. (12,14) are given in terms of three free parameters, such as the (dimensionless) longitudinal size $\Delta\eta$, the freeze-out temperature T and the effective temperature $T_* = T_e(y_0)$. Thus the longitudinal size $\Delta\eta$, which is hardly accessible to HBT measurements [2,10], can be determined from momentum space measurements alone!

The experimental analysis of LPTE is based on a comparison of the IMD in $p + A$ or $A + B$ reactions to the IMD in $p + p$ reaction at the same energy,

$$R_{pp}^{AB} = \frac{d^2 n_{A+B}}{dy\, dm_t^2} \left(\frac{d^2 n_{p+p}}{dy\, dm_t^2} \right)^{-1}. \tag{15}$$

This ratio can easily be evaluated from eq. (9) as

$$R_{pp}^{AB} = \exp\left(-\frac{m_t}{T_*^{AB}} + \frac{m_t}{T_*^{pp}} \right) \times$$
$$\exp\left(+ \frac{(y-y_0)^2 (\Delta^2\eta^{pp} - \Delta^2\eta^{AB})}{(\Delta^2\eta^{pp} + T/m_t)\,(\Delta^2\eta^{AB} + T/m_t)} \right) \tag{16}$$

If T is independent of the reaction type, R_{pp}^{AB} contains two factors. The first one is independent of the rapidity and may account for the various modifications of the p_t spectrum at mid-rapidity due to rescattering, nuclear resonance production etc. This factor may yield the LPTE at mid-rapidity. The second factor determines the rapidity dependence of the effect. Generally, the degree of stopping in $p+p$ reactions is smaller than in $p+A$ or $A+B$ reactions at the same energy, due to the enhanced rescattering in the second and third case. This also implies $\Delta\eta^{pp} > \Delta\eta^{pA} \geq \Delta\eta^{AB}$, consequently indicate an increase in R_{pp}^{AB} within the target and projectile fragmentation region. Thus the LPTE in $A+B$ or $p+A$ reactions as compared to $p+p$ reactions becomes a measure of the *difference* in the longitudinal sizes for the two reactions.

Can the LPTE be utilized to measure the *total* longitudinal source sizes (not only differences) ?

Let us introduce as an auxiliary quantity the ratio of the IMD-s for rapidities y_1 and y_2 at the same m_t *in the same reaction*, either $p+p$ or $p+A$ or $A+B$

$$R(y_1, y_2, m_t) = \frac{d^2n}{dy_1\, dm_t^2} \left(\frac{d^2n}{dy_2\, dm_t^2} \right)^{-1}. \tag{17}$$

The rapidity-independent part of the momentum-distribution, $g(m_t)$ cancels from this ratio:

$$R(y_1, y_2, m_t) = \frac{\int d\eta\, G(\eta)\, J_{m_t}(\eta, y_1)}{\int d\eta\, G(\eta)\, J_{m_t}(\eta, y_2)}. \tag{18}$$

Let us introduce the normalized ratio of the invariant momentum distributions,

$$L_R(y, m_t) = \frac{R(y, y_0, m_t)}{\lim\limits_{m_t \to \infty} R(y, y_0, m_t)} \tag{19}$$

where y_0 stands, as before, for the mid-rapidity. This normalized ratio shall be a measure of the LPTE as compared to the low-p_t distribution at mid-rapidity. The normalized ratio L_R has the property

$$\lim_{m_t \to \infty} L_R(y, m_t) = L_R(y_0, m_t) \equiv 1. \tag{20}$$

Although the limit $m_t \to \infty$ formally appears in the definition of L_R, the limiting value of $L_R(y, m_t)$ is 1, independently of the rapidity. This asymptotic value can be utilized for guiding the experimental normalization of this quantity. In the Gaussian approximation, L_R reads as

$$L_R(y, m_t) = \exp\left(+ \frac{(y - y_0)^2}{2\Delta\eta^2} \frac{T}{m_t\left(\Delta\eta^2 + T/m_t\right)} \right) \tag{21}$$

The LPTE becomes by eqs. (16,21) *exponentially enhanced* in the target and projectile fragmentation region. This is *due to the decrease of* $T_e(y)$ in the same region, cf. eq. (12). As a consequence, the low-p_t ratio $L_R(y, m_t)$ becomes a sensitive measure of both the geometrical size of the emission region in the longitudinal direction $\Delta\eta$ and the freeze-out temperature T. The LPTE has been calculated without explicit reference to the particle type, thus one might expect LPTE to exists for pions, kaons as well as for protons and any other heavier particles. At a given value of p_t, heavy particles have a substancially reduced LPTE compared to pions according to eq. (21).

The expression (21) indicates that the rapidity-dependent LPTE is a consequence of the finite size of the longitudinally expanding system, because the enhancement vanishes in the limit the longitudinal size goes to infinity,

$$\lim_{\Delta\eta \to \infty} L_R(y, m_t) = 1, \tag{22}$$

independently of y and m_t. Thus it is more natural to compare the IMD, at a given rapidity, to the IMD at mid-rapidity in the same reaction, than to compare the IMD in $p+A$ or $A+B$ reactions to the IMD in $p+p$ reaction, at the same energy. However, this latter possibility has been realized in most of the available data analysis on LPTE. As can be seen from eq. (16),

the rapidity dependence of the ratio R_{pp}^{AB} is also a consequence of the finite longitudinal sizes of the expanding systems since

$$\lim_{\substack{\Delta\eta^{pp},\Delta\eta^{AB}\to\infty, \\ (\Delta\eta^{pp}-\Delta\eta^{AB})<\infty}} R_{pp}^{AB} = \exp\left(-\frac{m_t}{T_*^{AB}} + \frac{m_t}{T_*^{pp}} \right), \tag{23}$$

which is rapidity independent.

The results presented here are insensitive to resonance decay effects if these are not coupled to a specific rapidity region, since in such cases the rapidity independent effective temperature, T_* may absorb their influence. A notable exception can be the delta resonance, which is copiously produced in the target and projectile fragmentation region. Since the effect of the longitudinal expansion is increasing with $y - y_0$ like an inverse Gaussian, eqs. (21,16), only a comparably strong coupling between the Δ production and the rapidity can influence it significantly. Such a strong coupling would be, however, rather unusual.

The result for the LPTE is insensitive to the presence of the transverse flow, since similar results can be obtained for the IMD of ref. [10], where the flow profile was approximated by a *three-dimensional* scaling flow which describes a fully developed transverse flow. The transverse flow cancels since it influences the transverse momentum spectra only at high values of m_t.

It is straightforward to generalize the above results for arbitrary $G(\eta)$ functions, to be published elsewhere.

Application. The E802 collaboration has found a characteristic bell-shaped curve for $T_e(y)$ in central $S + Au$ reactions at 14.6 AGeV [15]. It should be clear from the previous parts that the decrease of $T_e(y)$ with increasing $\mid y - y_0 \mid$ results in a rapidity-dependent LPTE. Thus a fit to $T_e(y)$ is a kind of measure of the rapidity-dependece ot LPTE. The E802 data are fitted with eqs. (12,13) in Fig. 1. The data are compatible with our description at a $\chi^2/NDF = 1.93$. This result is somewhat surprising since we have utilized the scaling flow profile to describe the longitudinal expansion. Note, however, that such a flow profile may

develop not only in reactions where the incoming nuclei pass through each other, but also may develop in the final stage of the hydrodynamical evolution in the stopping region [16]. The mid-rapidity at BNL AGS is at $y_{mid} \approx 1.7$ for symmetric reactions, the fit yields somewhat smaller value of $y_0 = 1.52 \pm 0.25$ for the asymmetric $S + Au$ collision. The mid-rapidity temperature is found to be $T_* = 209 \pm 10$ MeV, the freeze-out temperature cannot be determined precisely at the present level of the experimetal errors $T = 60 \pm 418$ MeV. Finally, we measure the length of the kaon-emitting volume at the freeze-out time, in dimensionless space-time rapidity units to be $\Delta\eta_K = 0.39 \pm 0.18$.

In summary, the model predicts a rapidity-dependent low-p_t enhancement for $p+p$, $p+A$ and $A + B$ reactions, when compared to the transverse mass spectrum at mid-rapidity in the same reactions. The relative enhancement in $p + A$ and $A + B$ reactions compared to the transverse momentum spectrum in $p + p$ reactions at the same energy turns out to be a consequence of the increased degree of stopping in $p + A$ and $A + B$ reactions as compared to the stopping in $p + p$ collisions.

The rapidity dependence of the low-p_t enhancement can be used to measure the finite longitudinal size for longitudinally expanding systems. This method is especially advantageous since the large longitudinal geometrical sizes of expanding systems appear in the radius parameters of Bose-Einstein correlation measurements as correction terms only [2,9,10]. Using a variation of this technique we have determined the length of the K^+ emitting volume at freeze-out in central $S + Au$ reactions at AGS to be $\Delta\eta_{K^+} = 0.39 \pm 0.18$ space-time rapidity unit.

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FIGURES

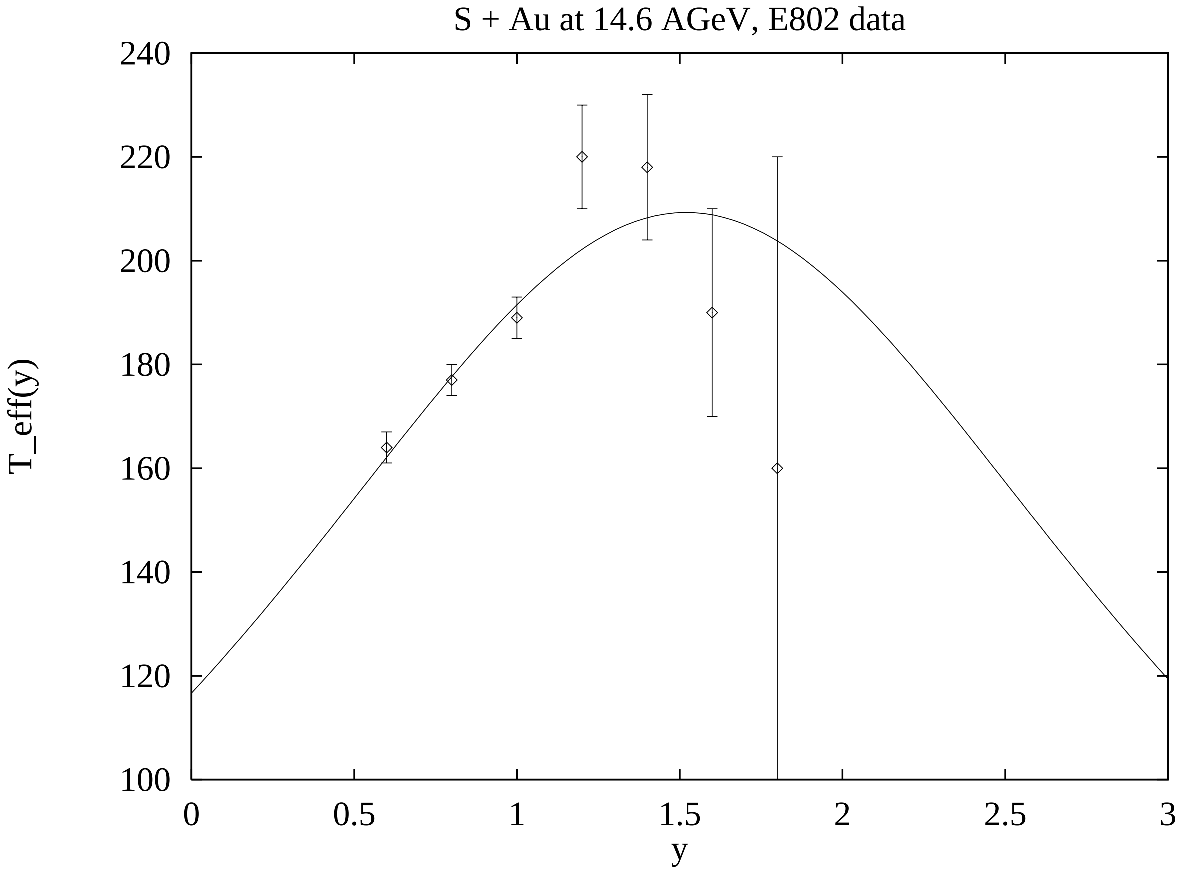

FIG. 1.

Fit to $T_e(y)$ of K^+ mesons in 14.6 AGeV $S + Au$ reactions with eqs. (12,13), solid line. Data points with error bars have been scanned from [15]. The fit parameters are $\Delta\eta = 0.39 \pm 0.18$, $T_* = 209 \pm 10$ MeV, $T = 60 \pm 418$ MeV and $y_0 = 1.52 \pm 0.25$ for the minimum of $\chi^2/NDF = 5.8/(7-4) = 1.93$ using $m_K = 494$ MeV.